Chapter 2

1

VECTORS

1

Lecture 02

2

- Component Method Of Adding Vectors
- Unit Vectors

2

Component Method of Adding Vectors

3

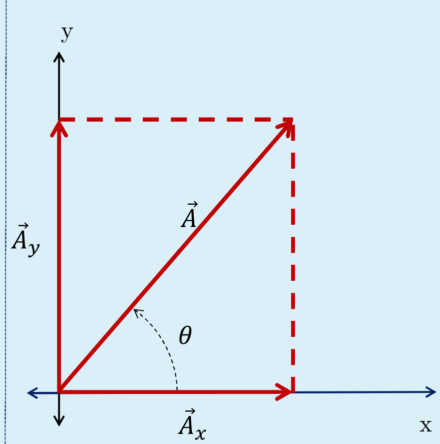
- Graphical addition is not recommended when:
  - High accuracy is required
  - If you have a three-dimensional problem
- Component method is an alternative method
  - It uses projections of vectors along coordinate axes

3

Components of a Vector, Introduction

4

- A **component** is a projection of a vector along an axis.
  - A_x : the component (projection) of the vector along the x-axis.
  - A_y : the component (projection) of the vector along the y-axis.



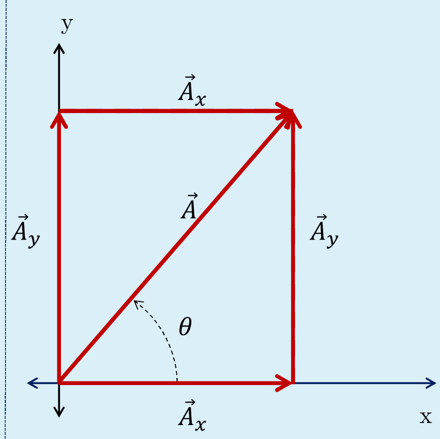
4

Components of a Vector, Introduction

5

$$\vec{A}_x + \vec{A}_y = \vec{A}$$

$$\vec{A}_y + \vec{A}_x = \vec{A}$$



5

Vector Component Terminology

6

- $\vec{A}_x$ and $\vec{A}_y$ are the component vectors of $\vec{A}$.
  - They are vectors and follow all the rules for vectors.
- $|\vec{A}_x|$ and $|\vec{A}_y|$ are scalars, and will be referred to as the components of $\vec{A}$.

6

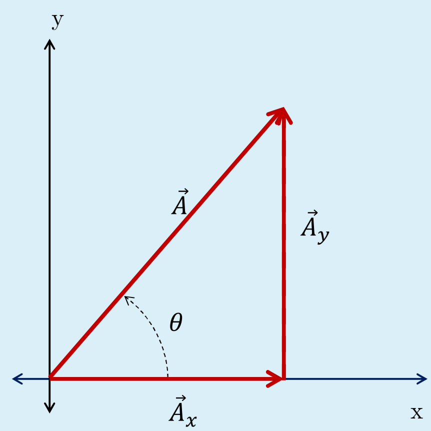
Components of a Vector

7

- Assume you are given a vector $\vec{A}$
- It can be expressed in terms of two other vectors, $\vec{A}_x$ and $\vec{A}_y$

$$\vec{A}_x + \vec{A}_y = \vec{A}$$

- These three vectors form a right triangle.



Mustafa Al-Zyout - Philadelphia University 29-Sep-25

7

Components of a Vector

8

The x-component is:

$$cos\theta = \frac{adjacent}{hypotenuse} = \frac{A_x}{A}$$

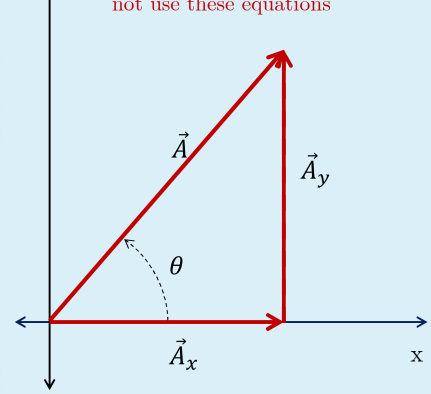
$$A_x = A\ cos\theta$$

The y-component is:

$$sin\theta = \frac{opposite}{hypotenuse} = \frac{A_y}{A}$$

$$A_y = A\ sin\theta$$

The angle θ is measured with respect to the x-axis. IF NOT, do not use these equations



Mustafa Al-Zyout - Philadelphia University 29-Sep-25

8

Components of a Vector

9

The magnitude of the vector is:

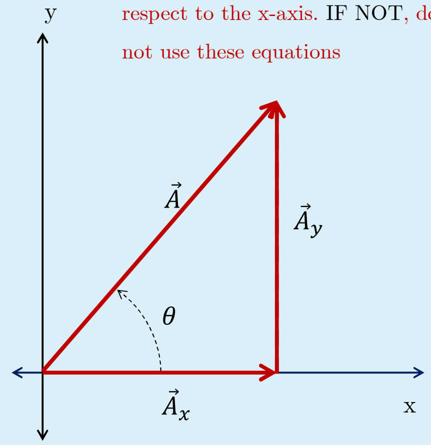
$$A = \sqrt{A_x^2 + A_y^2}$$

The direction of the vector is:

$$tan\theta = \frac{opposite}{adjacent} = \frac{A_y}{A_x}$$

$$\theta = tan^{-1}\frac{A_y}{A_x}$$

The angle θ is measured with respect to the x-axis. IF NOT, do not use these equations

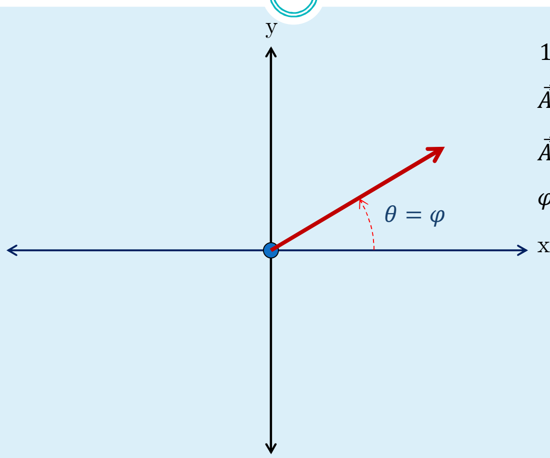


Mustafa Al-Zyout - Philadelphia University 29-Sep-25

9

Direction (θ) and calculations from your calculator (φ)

10



$1st\ quadrant$

$\vec{A}_x$: +ve

$\vec{A}_y$: +ve

φ : +ve

Mustafa Al-Zyout - Philadelphia University 29-Sep-25

10

Direction (θ) and calculations from your calculator (φ)

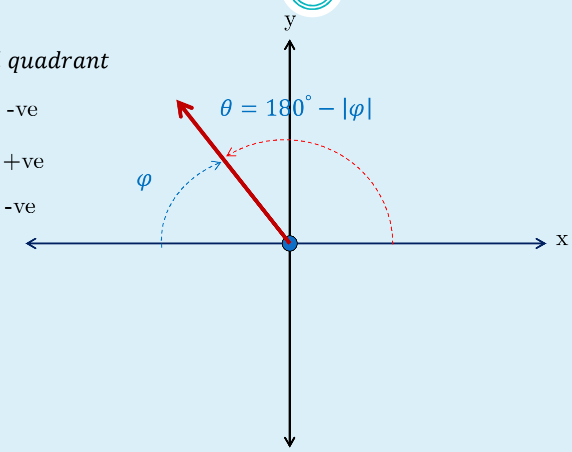
11

2nd quadrant

$\vec{A}_x$: -ve

$\vec{A}_y$: +ve

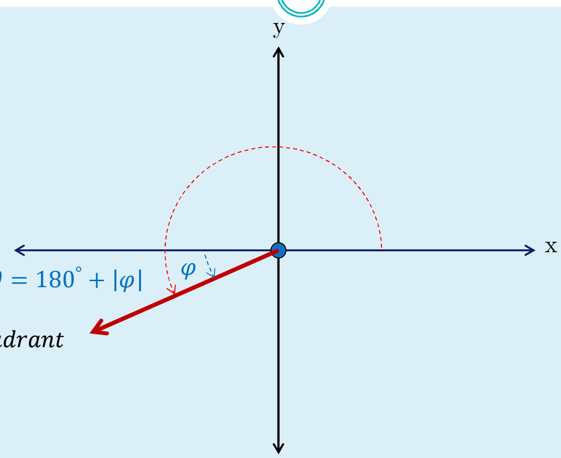
φ : -ve



11

Direction (θ) and calculations from your calculator (φ)

12



3rd quadrant

$\vec{A}_x$: -ve

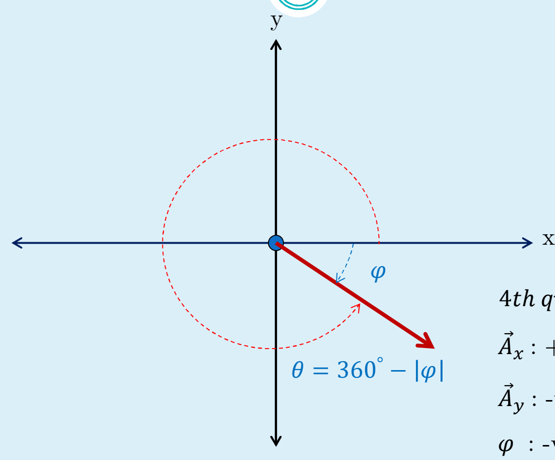
$\vec{A}_y$: -ve

φ : +ve

12

Direction (θ) and calculations from your calculator (φ)

13



4th quadrant

$\vec{A}_x$: +ve

$\vec{A}_y$: -ve

φ : -ve

13

Unit Vectors

14

- A unit vector is a dimensionless vector with a magnitude of exactly 1.
- Unit vectors are used to specify a direction and have no other physical significance.

14

Unit Vectors, cont.

15

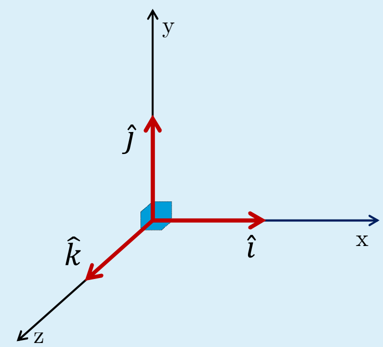
- The symbols $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ represent unit vectors.
- The magnitude of each unit vector is 1.

$$\hat{\imath} = \hat{\jmath} = \hat{k} = 1$$

- They form a set of mutually perpendicular vectors in a right-handed coordinate system.

$$\hat{\imath} \perp \hat{\jmath} \perp \hat{k}$$

- Dimensionless vectors.



15

Unit Vectors in Vector Notation

16

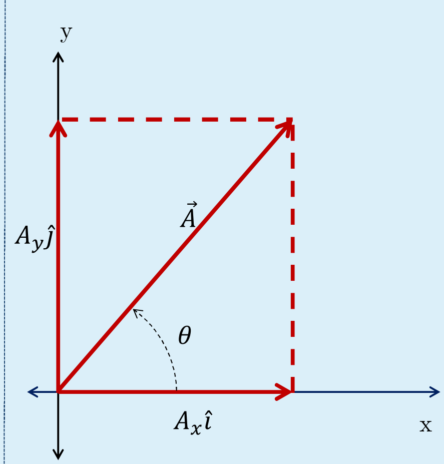
- $\vec{A}_x$ is the same as $A_x\hat{\imath}$, $\vec{A}_y$ is the same as $A_y\hat{\jmath}$ and $\vec{A}_z$ is the same as $A_z\hat{k}$.
- The complete vector can be expressed as:

$$\vec{A} = |\vec{A}|, \theta$$

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

$$\vec{A} = A\cos\theta\hat{\imath} + A\sin\theta\hat{\jmath}$$



16

Position Vector, Example

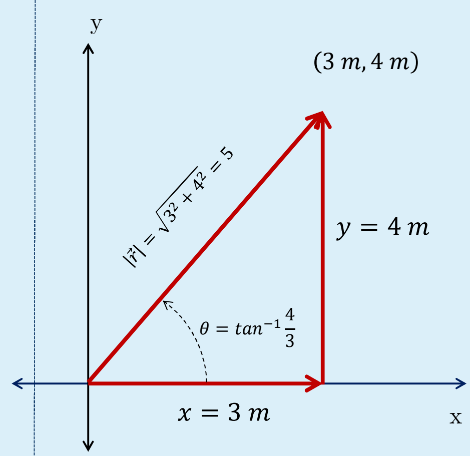
17

- If $x = 3m$ and $y = 4m$:

$\vec{r} = 3 + 4 = 7$ X

$\vec{r} = \vec{3}_x + \vec{4}_y$?

$\vec{r} = 3\hat{\imath} + 4\hat{\jmath}$ √



Mustafa Al-Zyout - Philadelphia University 29-Sep-25

17

Adding Vectors Using Unit Vectors

18

- Given:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

- Find: $\vec{R} = \vec{A} + \vec{B}$

$$\vec{R} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$

$$\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath}$$

Mustafa Al-Zyout - Philadelphia University 29-Sep-25

18

Adding Vectors Using Unit Vectors

19

So, the x-component of the resultant is:

$$R_x = A_x + B_x$$

The y-component of the resultant is:

$$R_y = A_y + B_y$$

The magnitude of the resultant is:

$$R = \sqrt{R_x^2 + R_y^2}$$

The direction of the resultant is:

$$\theta = tan^{-1}\frac{R_y}{R_x}$$

Mustafa Al-Zyout - Philadelphia University 29-Sep-25

19

Three-Dimensional Extension

20

- Given:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

- Find: $\vec{R} = \vec{A} + \vec{B}$

$$\vec{R} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$

$$\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k}$$

Mustafa Al-Zyout - Philadelphia University 29-Sep-25

20

Three-Dimensional Extension

21

So, the x-component of the resultant is:

$$R_x = A_x + B_x$$

The y-component of the resultant is:

$$R_y = A_y + B_y$$

The z-component of the resultant is:

$$R_z = A_z + B_z$$

The magnitude of the resultant is:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

The direction of the resultant is:

$$LATER$$

21

Adding Three or More Vectors

22

- The same method can be extended to adding three or more vectors.
- Assume

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\vec{C} = C_x\hat{\imath} + C_y\hat{\jmath} + C_z\hat{k}$$

- And

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

$$\vec{R} = (A_x + B_x + C_x)\hat{\imath} + (A_y + B_y + C_y)\hat{\jmath} + (A_z + B_z + C_z)\hat{k}$$

22

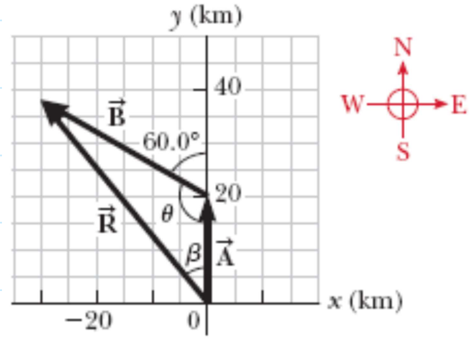
Using unit vectors - 1

Sunday, 13 June, 2021 15:08

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A car travels $20\ km$ due north and then $35\ km$ in a direction 60° west of north. Find the magnitude and direction of the car's resultant displacement.



Solution:

Let:

$\vec{A}$ represents the first displacement, in magnitude - direction notation:

$$\vec{A} = [20\ km, 90°]$$

$\vec{B}$ represents the second displacement, in magnitude - direction notation:

$$\vec{B} = [35\ km, 150°]$$

and $\vec{R}$ represents the resultant displacement:

$$\vec{R} = \vec{A} + \vec{B}$$

Find the components of $\vec{A}$

$$A_x = A\cos\theta_A = 20\cos(90°) = 0\ km$$

$$A_y = A\sin\theta_A = 20\sin(90°) = 20\ km$$

Find the components of $\vec{B}$

$$B_x = B\cos\theta_B = 35\cos(150°) = -30.3\ km$$

$$B_y = B\sin\theta_B = 35\sin(150°) = 17.5\ km$$

Write $\vec{A}$ and $\vec{B}$ in unit vector notation:

$$\vec{A} = (0\hat{\imath} + 20\hat{\jmath})\ km$$

$$\vec{B} = (-30.3\hat{\imath} + 17.5\hat{\jmath})\ km$$

Write the total displacement in unit vector notation:

$$\vec{R} = \vec{A} + \vec{B} = \left((0 - 30.3)\hat{\imath} + (20 + 17.5)\hat{\jmath}\right)\, km$$

$$\vec{R} = (-30.3\hat{\imath} + 37.5\hat{\jmath})\, km$$

The magnitude of $\vec{R}$:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-30.3)^2 + (37.5)^2} = 48.2\, km$$

The direction of $\vec{R}$:

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{37.5}{-30.3}\right) \cong -51$$

Since the sign of R_x is negative and the sign of R_y is positive, the resultant displacement lies in the second quadrant of the coordinate system. That is,

$$\theta = 180° - |\varphi| = 180° - 51 = 129°$$

The resultant displacement of the car in magnitude - direction notation is:

$$\vec{R} = [48.2\, km, 129°]$$

Using unit vectors - 2

Friday, 29 January, 2021 21:33

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Find the sum of two displacement vectors $\vec{A}$ and $\vec{B}$ lying in the xy plane and given by: $\vec{A} = (2\hat{\imath} + 2\hat{\jmath})\ m$ and $\vec{B} = (2\hat{\imath} - 4\hat{\jmath})\ m$.

Solution

Comparing this expression for $\vec{A}$ with the general expression $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$, we see that:

$$A_x = 2\ m, A_y = 2\ m, \text{ and } A_z = 0.$$

Likewise,

$$B_x = 2\ m, B_y = 4\ m, \text{ and } B_z = 0.$$

We can use a two-dimensional approach because there are no z components.

$$\vec{R} = \vec{A} + \vec{B} = (2+2)\hat{\imath} + (2-4)\hat{\jmath} = 4\hat{\imath} - 2\hat{\jmath}$$

the components of $\vec{R}$:

$$R_x = 4\ m\ ;\ R_y = -2\ m$$

Since the sign of R_x is positive and the sign of R_y is negative, the resultant displacement lies in the fourth quadrant of the coordinate system.

the magnitude of $\vec{R}$:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4)^2 + (-2)^2} = \sqrt{20}\ m = 4.5\ m$$

the direction of $\vec{R}$:

$$\tan\theta = \frac{R_y}{R_x} = \frac{-2}{4} = -0.5$$

Your calculator likely gives the answer:

$$(-27°) \text{ for } \theta = \tan^{-1}(-0.5).$$

This answer is correct if we interpret it to mean $(27°)$ clockwise from the x axis. Our standard form has been to quote the angles measured counterclockwise from the $+x$ axis, and that angle for this vector is:

$$\theta = 360° - 27° = 333°.$$

Using unit vectors - 3

Friday, 29 January, 2021 21:33

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A particle undergoes three consecutive displacements: $\Delta\vec{r}_1 = \left(15\hat{\imath} + 30\hat{\jmath} + 12\hat{k}\right) cm$, $\Delta\vec{r}_2 = \left(23\hat{\imath} - 14\hat{\jmath} - 5\hat{k}\right) cm$ and $\Delta\vec{r}_3 = (-13\hat{\imath} + 15\hat{\jmath})\ cm$. Find unit-vector notation for the resultant displacement and its magnitude.

Solution

To find the resultant displacement, add the three vectors:

$$\Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3$$

$$= (15 + 23 - 13)\hat{\imath} + (30 - 14 + 15)\hat{\jmath} + (12 - 5.0 + 0)\hat{k}$$

$$= \left(25\hat{\imath} + 31\hat{\jmath} + 7.0\hat{k}\right) cm$$

Find the magnitude of the resultant vector:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(25)^2 + (31)^2 + (7)^2} = 40\ cm$$

Using unit vectors - 4

Friday, 29 January, 2021 21:33

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.

H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Given the two displacements: $\vec{A} = \left(15\hat{\imath} + 30\hat{\jmath} + 12\hat{k}\right)\,cm$ and $\vec{B} = \left(23\hat{\imath} - 14\hat{\jmath} - 5\hat{k}\right)\,cm$. Find the magnitude of the displacement $2\vec{A} - \vec{B}$.

Solution

$$2\vec{A} - \vec{B} = 2\left(15\hat{\imath} + 30\hat{\jmath} + 12\hat{k}\right) - \left(23\hat{\imath} - 14\hat{\jmath} - 5\hat{k}\right)$$

$$= \left(30\hat{\imath} + 60\hat{\jmath} + 24\hat{k}\right) - \left(23\hat{\imath} - 14\hat{\jmath} - 5\hat{k}\right)$$

$$= \left(7\hat{\imath} + 74\hat{\jmath} + 29\hat{k}\right)\,cm$$

Find the magnitude of the resultant vector:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(7)^2 + (74)^2 + (29)^2} = 79.8\,cm$$